1. A die will be rolled some number of times.

(a) (3 pts) You win $3 if the die shows $\geq 0.19$ of the time. Which do you prefer, 40 rolls or 400 rolls, or do both options offer the same chance of winning? Explain your answer.

**Solution:** The more you roll the die the better the change that the percentage of $\geq 0.19$’s will be close to 16.666%. Since $19\% > 16.666\%$, the more you roll the die the less likely it is that you see more than 19%, so in this case you are better off with 40 rolls.

(b) (3 pts) You win $3 if the die shows $\geq 0.13$ of the time. Which do you prefer, 40 rolls or 400 rolls, or do both options offer the same chance of winning? Explain your answer.

**Solution:** As above, the more you roll the die, the more likely it is that the percentage of $\geq 0.13$’s will be close to 16.666% (= 1/6). Since 16.666% > 13%, it follows that the more you roll the die, the more likely it is to win in this scenario, so 400 rolls is better here.

2. Nine hundred tickets are drawn at random with replacement from the box

$$\begin{array}{cccccc}
0 & 0 & 1 & 1 & 4 & 6
\end{array}$$

(a) (2 pts) What is the expected value of the sum of the draws?

**Solution:** The average of the box is

$$Avg = \frac{0 + 0 + 1 + 1 + 4 + 6}{6} = 2$$

so the Expected Value for the sum of the draws is

$$EV = n \cdot Avg = 900 \cdot 2 = 1800.$$  

(b) (2 pts) What is the standard error for the sum of the draws? (Round your answer to the nearest integer.)

**Solution:** The standard deviation of the box is

$$SD = \sqrt{\frac{(0-2)^2 + (0-2)^2 + (1-2)^2 + (1-2)^2 + (4-2)^2 + (6-2)^2}{6}} = \sqrt{30/6} = \sqrt{5}$$

so the standard error for the sum of the draws is

$$SE = \sqrt{n} \cdot SD = \sqrt{900} \cdot \sqrt{5} \approx 67.$$  

(c) (2 pts) What is the probability that the **Average** of the draws is less than 2.045?

**Solution:** Recall that $Avg = \frac{\text{Sum}}{n}$ and $\text{Sum} = n \cdot Avg$, so the average of the draws (in this case) is greater than 2.045 exactly when the sum of the draws is greater than $900 \cdot 2.045 = 1840.5$. To estimate this probability, we use the normal approximation (*as described in chapter 17*).

First, convert 1840.5 to standard units using the EV and the SE:

$$\frac{1840.5 - 1800}{67} \approx 0.6 \text{ standard units}$$

which means that

$$P(Avg \geq 2.045) = P(\text{Sum} \geq 1840.5) \approx \text{area under normal curve to the right of 0.6}.$$  

From the normal table, the area between −0.6 and 0.6 is 45.15%, and the area to the right of 0.6 is therefore 50% − (45.15%/2), so

$$P(Avg \geq 2.045) = P(\text{Sum} \geq 1840.5) \approx 50\% - 22.575\% \approx 27.4\%.$$
3. Consider the box [1 2 2 5]

(a) (2 pts) If 3 tickets are drawn from this box at random \textit{without} replacement, what is the probability that the sum of the draws is 8?

\textbf{Solution:} The only way to get a sum of 8 with three draws \textit{without replacement} is with the combination [5 2 1] \textit{in some order}. There are 6 different ways that the sequence (1, 2, 5) can occur, and each of these sequences can occur twice, because there are two 2\'s to choose from, so there are a total of 12 draws that produce a sum of 8.

Now, there are a total of 24 = 4 \cdot 3 \cdot 2 different combinations of three tickets that can be drawn from the box \textit{without replacement}, so

\[ P(\text{sum of three draws without replacement} = 8) = \frac{12}{24} = \frac{1}{2}. \]

(b) (2 pts) If 3 tickets are drawn from this box at random \textit{with} replacement, what is the probability that the sum of the draws is 8?

\textbf{Solution:} Drawing with replacement does not change the number of ways that you can get a sum of 8, since you still need [5 2 1] \textit{in some order}, and there are still exactly 12 ways to do this. On the other hand, there are now a total of 64 = 4 \cdot 4 \cdot 4 different possible triples of tickets that can be drawn, so

\[ P(\text{sum of three draws with replacement} = 8) = \frac{12}{64} = \frac{3}{16}. \]

(c) (2 pts) If 3 tickets are drawn from this box at random \textit{with} replacement, what is the probability that \textit{at least} one of the tickets is a 1?

\textbf{Solution:} The probability that \textit{no} 1\'s are observed in 3 draws (with replacement) is

\[ P(\text{no 1\'s in three draws}) = \left( \frac{3}{4} \right)^3 = \frac{27}{64} \]

so the probability that at least one 1 is observed is

\[ P(\text{at least one 1 in three draws}) = 1 - P(\text{no 1\'s in three draws}) = 1 - \frac{27}{64} = \frac{37}{64}. \]

4. A study comprising several quarters at Small State College investigated the relation between students’ midterm exam scores and final exam scores in the course Psychology 1A. The data is summarized with the statistics below:

\begin{align*}
\text{Average on midterm} &= 75, \quad \text{SD midterm} = 10 \\
\text{Average on final} &= 65, \quad \text{SD final} = 14, \quad r = 0.8.
\end{align*}

(a) (3 pts) Find the \textit{regression equation} for predicting final exam score from midterm exam score.

\textbf{Solution:} The slope coefficient is

\[ \beta_1 = r \cdot \frac{SD_F}{SD_M} = 0.8 \cdot \frac{14}{10} = 1.12 \]

and the constant coefficient is

\[ \beta_0 = \text{Avg}_F - \beta_1 \cdot \text{Avg}_M = 65 - 1.12 \cdot 75 = -19, \]

so the regression equation for predicting the score on the final from the score on the midterm is

\[ F = -19 + 1.12 \cdot M. \]
(b) (2 pts) Find the standard error of regression for predicting final exam score from midterm exam score.

Solution: The standard error of regression for predicting Final score from Midterm score is

\[ SER = \sqrt{1 - r^2} \cdot SD_F = \sqrt{1 - 0.64} \cdot 14 = 8.4. \]

(c) (2 pts) What is the predicted score on the final of a student who scored 70 on the midterm? Include a give-or-take number.

Solution: Use the regression equation to predict the score and the SER for the give-or-take number:

\[ F(70) = (-19 + 1.12 \cdot 70) \pm 8.4 = 59.4 \pm 8.4. \]